

Solving Linear Systems of Equations (Graphing & Substitution Methods)

Problem 1: Systems of Linear Equations

Ramona sets up a lemonade stand in front of her house. Each cup of lemonade costs Ramona \$0.30 to make, and she spends \$6 on the advertising signs she puts up around her neighborhood. She sells each cup of lemonade for \$1.50.

Define the variables:

x will represent the number of cups of lemonade sold.

y will represent the amount of money earned or spent (in dollars).

1. Write an equation that describes Ramona's income.

$$y = 1.50x$$

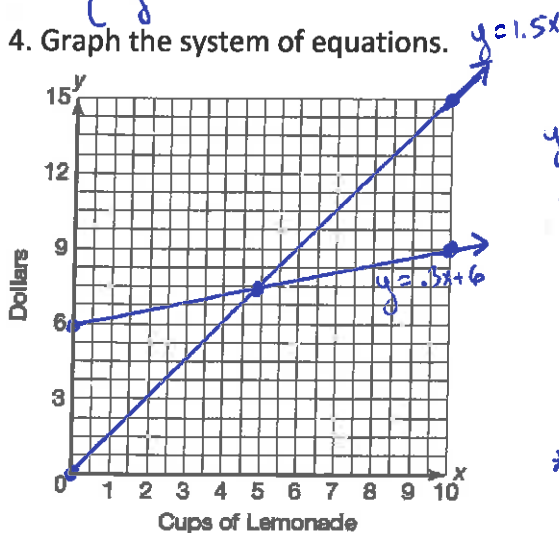
2. Write an equation that describes Ramona's expenses.

$$y = .30x + 6$$

3. Write the **System of Equations** that represents Ramona's lemonade sales.

$$\begin{cases} y = 1.5x \\ y = .3x + 6 \end{cases}$$

4. Graph the system of equations.



$$\begin{aligned} y &= 1.5x \\ y\text{-int} &= 0 \quad (0,0) \\ y &= 1.5(10) \\ y &= 15 \\ &(10,15) \end{aligned}$$

$$\begin{aligned} y &= .3x + 6 \\ y\text{-int} &= 6 \quad (0,6) \\ y &= .3(10) + 6 \\ &= 3 + 6 \\ &= 9 \\ &(10,9) \end{aligned}$$

*plot two points for each line.

5. The **Break-Even point** is the point when the costs and income are equal. Estimate the break-even point on the graph. Explain what the break-even point means in the context of the problem situation.

Break even point is 5 cups of lemonade.
If Ramona sells 6 cups of lemonade, she will start to make a profit.

Problem 2:

Olivia is building birdhouses to raise money for a trip to Hawaii. She spends a total of \$30 on the tools needed to build the houses. The material to build each birdhouse costs \$3.25. Olivia sells each birdhouse for \$10.

1. Define two variables for this problem situation.

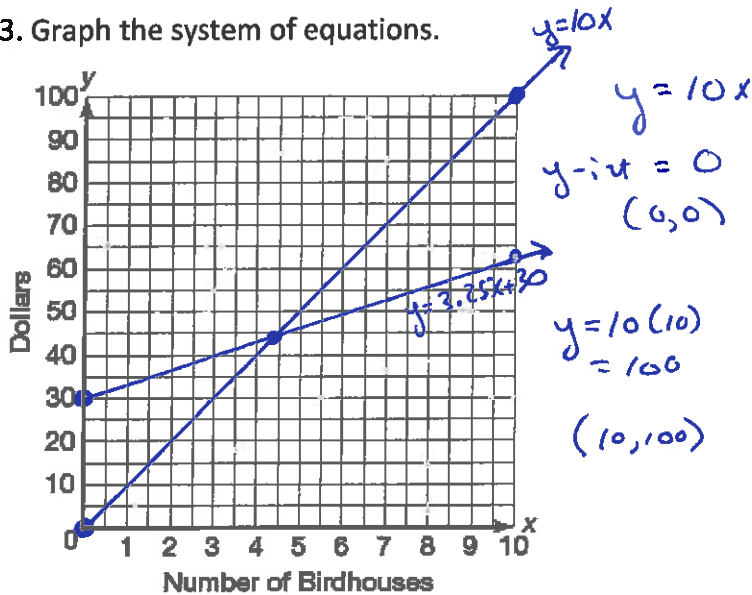
$$x = \# \text{ of bird houses made}$$
$$y = \text{money earned/spent.}$$

2. Write a system of equations that represent Olivia's income and expenses. Use the variables defined above.

$$y = 10x \quad : \text{income from selling bird houses.}$$

$$y = 3.25x + 30 \quad : \text{Expenses for selling bird houses.}$$

3. Graph the system of equations.



$$y = 3.25x + 30$$
$$y\text{-int} = 30 \quad (0, 30)$$

$$y = 3.25(10) + 30$$
$$= 32.5 + 30$$
$$= 62.5$$
$$(10, 62.5)$$

4. Estimate the break-even point and explain its meaning in the context of the situation.

the break even point is between 4 and 5 bird houses.

She must sell more than 4 houses to make a profit.

Problem 3: Solving a Linear System by Substitution

Solve the System: $\begin{cases} y = 1.5x \\ y = 0.3x + 6 \end{cases}$

1. Substitute the y from one equation in for the y in the other equation.

$$1.5x = 0.3x + 6$$

2. Solve for x.

$$\begin{array}{r} 1.5x = 0.3x + 6 \\ -0.3x \quad -0.3x \\ \hline 1.2x = 6 \\ 1.2 \quad 1.2 \quad x = 5 \end{array}$$

3. Solve for y.

$$\begin{aligned} y &= 1.5(5) \\ &= 7.5 \end{aligned}$$

4. Write answer as an ordered pair.

$$(5, 7.5)$$

Solve each system:

1. $\begin{cases} 8x = 2y + 7 \\ y = -2x + 4 \end{cases}$

$$8x = 2(-2x + 4) + 7$$

$$8x = -4x + 8 + 7$$

$$\begin{array}{r} 8x = -4x + 15 \\ +4x \quad +4x \\ \hline 12x = 15 \end{array}$$

$$\begin{aligned} 12x &= 15 \\ x &= 1.25 \end{aligned}$$

$$\begin{aligned} y &= -2(1.25) + 4 \\ &= 1.5 \end{aligned}$$

$$(1.25, 1.5)$$

2. $\begin{cases} \frac{1}{2}x + \frac{1}{4}y = 6 \\ y = 4 \end{cases} \Rightarrow \begin{cases} 2x + y = 24 \\ y = 4 \end{cases}$

$$\begin{array}{r} 2x + y = 24 \\ -4 \quad -4 \\ \hline 2x = 20 \\ x = 10 \end{array}$$

$$y = 4$$

$$(10, 4)$$

3. $\begin{cases} 0.4x + 0.3y = 1 \\ -0.2x + 0.1y = 0 \end{cases} \Rightarrow \begin{cases} 4x + 3y = 10 \\ -2x + y = 0 \end{cases}$

$$\begin{aligned} -2x + y &= 0 \\ y &= 2x \end{aligned}$$

$$4x + 3(2x) = 10$$

$$4x + 6x = 10$$

$$10x = 10$$

$$x = 1$$

$$y = 2(1)$$

$$y = 2$$

$$(1, 2)$$

Problem 4: Consistent & Inconsistent Systems

Consistent System: A system that has one or many solutions.

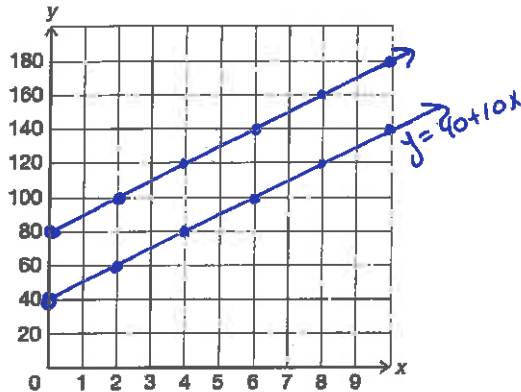
Inconsistent System: A system that has no solution.

1. Consider the system:
$$\begin{cases} y = 80 + 10x \\ y = 40 + 10x \end{cases}$$

$$\begin{aligned} y &= 80 + 10x \\ y\text{-int} &= 80 \\ m &= 10 \end{aligned}$$

$$\begin{aligned} y &= 40 + 10x \\ y\text{-int} &= 40 \\ m &= 10 \end{aligned}$$

a. Graph the system. Use the graph to determine the solution.



b. Solve the system using substitution. Does the solution agree with the graph? Explain.

$$\begin{array}{r} 40 + 10x = 80 + 10x \\ -10x \quad -10x \\ \hline 40 = 80 \leftarrow \text{false} \end{array}$$

there is no solution. this agrees with the graph because the lines don't intersect.

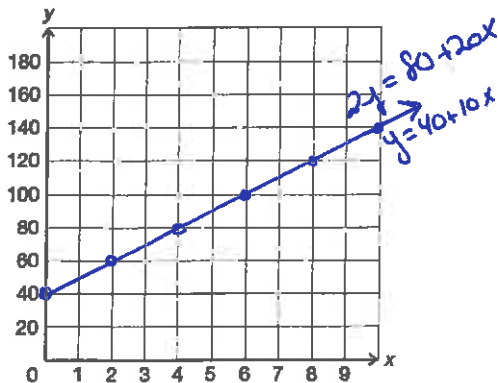
c. Is this system Consistent or Inconsistent? Explain.

Inconsistent because it has no solution.

2. Consider the system:
$$\begin{cases} 2y = 80 + 20x \\ y = 40 + 10x \end{cases}$$

$$\begin{aligned} \frac{2y}{2} &= \frac{80 + 20x}{2} \\ y &= 40 + 10x \end{aligned}$$

a. Graph the system. Use the graph to determine the solution.



b. Solve the system using substitution. Does the solution agree with the graph? Explain.

$$\begin{array}{r} 2(40 + 10x) = 80 + 20x \\ 80 + 20x = 80 + 20x \\ -20x \quad -20x \\ \hline 80 = 80 \leftarrow \text{true} \end{array}$$

there are an infinite # of solutions. this agrees with the graph because the two lines lie on top of each other.

c. Is this system Consistent or Inconsistent? Explain.

Consistent, because it has many solutions.